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Research article

A Computational Analysis of Convection Heat Transfer

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Abstract

Traditionally, in science and engineering the computer simulation has been applied mostly to obtain numerical solutions. Moreover, in the last decade the availability of increasing powerful computer has been introduced the possibility of obtaining analytical solutions by using algebraic manipulators. On the other hand, software like Mathematica have opened the door to the solution of complex engineering.

Key words: Hybrid analysis, generalized integral transform technique, Nusselt number, parallel plate channel, Mathematica, hybrid methods in engineering.

1 Introduction

In this paper we studied in numerical computation which is completely dominated the science and engineering. But over the past decade powerful computer systems have been widely available to carry out hybrid computation.

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[1, 2]. This software opens a new avenue for solving complicated engineering problems using hybrid analysis that combine analytical and numerical approaches. A full revision of the existing engineering methods is needed, such software may provide to dominate the hybrid methods in the future. Now, this text is created by using original notebooks documents [3], where all steps of the solutions- statements of the problem, nondimensionalizing, eigenproblem, integral transform pair, transforming to ordinary differential equations, solutions, computational rules, and results are made by using *Mathematica* [1]. Furthermore, notebooks can be used not to read but also to execute the input statements, plot graphics and animation. The goal of the present paper is to demonstrate that the generalized integral transform technique (GITT), which is a typical hybrid method could be efficiently used to solve heat convection problems with *Mathematica* and for the latest development in the GITT [4,7].

Hence considering the classical Graetz problems of steady state heat transfer in thermally developing , hydrodynamically developed forced laminar flow inside a parallel plate channel. The exact solution in terms of the Kummer confluent hypergeometric function is presented.

2 Exact Solution for Tube Flow

We consider the classical Graetz problem of steady-state heat transfer in thermally developing, hydro-dynamically developed force laminar flow inside a tube under,

The dimensionless temperature $\theta[\xi,\eta]$ of a fluid along a channel $0\leq \eta \leq \infty$

in the region $0\leq \xi \leq 1$ [5]

$$w[\xi]\frac{\partial\,\theta[\xi,\eta]}{\partial\eta} = \frac{\partial^2\,\theta[\xi,\eta]}{\partial\xi^2} + \frac{1}{\xi}\,\frac{\partial\,\theta[\xi,\eta]}{\partial\xi} \qquad (1)$$

$$\left(\xi\,\frac{\partial\theta}{\partial\eta}[\xi,\eta]\right)_{\xi\to 0} = 0, \theta[1,\eta] = 0, \theta[\xi,0] = 1 \qquad (2)$$

The velocity profile for laminar flow is [5]

$$w[\xi] = 2\,(1-\xi^2) \qquad (3)$$

Once $\theta[\xi,\eta]$ are determined the average temperature $\theta av[\eta]$ and Nusselt number Nu $[\eta]$ are evaluated from [4]

$$\theta av\,[\eta] == \frac{\int_0^1 \xi w[\xi]\theta\,[\xi,\eta]d\xi}{\int_0^1 \xi w[\xi]d\xi} \qquad (4)$$

$$Nu[\eta] = -\frac{\frac{\partial\theta\,[1,\eta]}{\partial\xi}}{eav[\eta]} \qquad (5)$$

To solve the problem by Eq. (1) we use the following eigenvalue problem,

$$\psi_i[\xi] + \frac{1}{\xi}\psi_i'[\xi] + \mu_i^2 w\,[\xi]\psi_i[\xi] = 0 \qquad (6)$$

$$(\xi\,\psi_i'[\,\xi])_{\xi\to 0} \qquad (7)$$

$$\psi_i[1] == 0 \qquad (8)$$

with the normalization condition then we obtain

$$\int_0^1 \xi\, w\,[\xi]\psi_i\,[\xi]^2 d\xi = 1 \qquad (9)$$

The solution of $\theta[\xi,\eta]$ has the form

$$\theta[\xi,\eta] == \sum_{i=1}^{n}\theta_i\,[\eta]\psi_i[\xi] \qquad (10)$$

where the summation is taken to the truncation number *n* and

$\theta_i[\eta] == \int_o^1 \xi\, w[\xi]\theta\,[\xi,\eta]\psi_i\,[\xi]d\xi$ (11)

Eq.(11) is the finite integral transform of the function $\theta[\xi,\eta]$ with respect to the space variable ξ and Eq.(10) is the inversion formula of the finite integral transform eqs. (11). Now, to transform the Eq.(1) into an ordinary differential equation for the integral transform we use the identity,

$$\int_o^1 \xi\,\psi_i\,[\xi]\frac{\partial^2\theta\,[\xi,\eta]}{\partial\xi^2}d\xi - \int_o^1 \xi\,\theta\,[\xi,\eta]\psi_i[\xi]d\xi +$$

$$\int_0^1 \psi_i\,[\xi]\frac{\partial\theta\,[\xi,\eta]}{\partial\xi}d\xi - \int_o^1 \theta\,[\xi,\eta]\,\psi_i'[\xi]d\xi =$$

$$-\theta[1,\eta]\psi_i'\,[1]+\psi_i\,[1]\frac{\partial\theta}{\partial\xi}[1,\eta] \quad (12)$$

From Eqs. (1)-(2) we find $''\partial''^2\,\theta\,[\xi,\eta]/''\partial\xi''^2$ and $\theta[\,1,\eta]$. From Eqs.(6) and (8) we find $\psi_i\,[\xi] and\, \psi_i\,[1]$. The results are introduced into Eq. (12) to obtain

$\theta_i'[\eta] + \mu_i^2\theta_i\,[\eta] = 0$ (13)

The entrance condition Eq.(2) is transformed as

$\theta_i[0] = -\frac{\psi_i'}{\mu_i^2}[1]$ (14)

The solution of Eqs (13) and (14) substituted into Eq.(10) gives the temperature

$\theta[\xi,\eta] = -\sum_{i=1}^n \frac{\psi_i'}{\mu_i^2}[1]\psi_i[\xi]e^{-\eta\mu_i^2}$ (15)

To obtain the average temperature we substituted Eq.(15) into Eq.(4)

$\theta av\,[\eta] = \sum_{i=1}^n \frac{\mu_i'}{\mu_i^4}[1]^2\frac{e^{-\eta\mu_i^2}}{\int_0^1 \xi\, w\,[\xi]d\xi}$ (16)

To obtain the Nusselt number we substitute eqs (15) and (16) into Eq.(5) then

$Nu[\eta] = \int_0^1 \xi w[\xi]d\xi\, \sum_{i=1}^n \frac{\mu_i'}{\mu_i^2}\,[1]^2 e^{-\eta\mu_i^2}$ (17)

The limiting Nusselt number is

$Nu[\infty] = \mu_1^2\,\int_0^1 \xi w[\xi]d\xi$ (18)

Introducing $\nu_i^2 = 2\,\mu_i^2$ and Eq.(3) into eqn. (6) we obtain

$\psi_i + \frac{1}{\xi}\,\psi_i'[\,\xi] + \nu_i^2(1-\xi^2)\psi_i[\xi] = 0$ (19)

The exact solution of Eqs. (6) and (7)

$\psi_i[\xi]$= C Hypergeometric 1F1 $\left[\frac{2-\nu[i]}{4},\ 1,\ \nu[i]\xi^2\right]\ Exp\,[-\frac{\nu[i]}{2}\,\xi^2]$ (20)

Where Hypergeometric 1F1 [a, b, z] represent s the Kummer confluent hypergeometric function [1].

Eq. (8) gives the eigencondition

Hypergeometric1F1 $\left[\frac{1}{4}\,(2-\nu_i), 1,\ \nu_i\,\right] = 0$

(21)

The normalization condition eq. (9) gives

$$C[i]^2 = \frac{1}{2\int_0^1 e^{-\xi^{2\nu_i}}\xi\,(1-\xi^2)\,Hypergeometric\,1F1\left[\frac{1}{4}(2-\nu_{i)},\ 1,\xi^2\,\nu_i\right]\left[\frac{1}{4}(2-\nu_{i)},\ 1,\xi^2\,\nu_i\right]^2 d\xi} \quad (22)$$

3 Rules and Computations

The *Mathematica* rules for the exact solution obtained and the temperature is given by Eq. (15) is programmed as

$\theta\ [-, o., -] := 1$

$\theta\ [1|1., -, -] := 0|\ ]$

$\theta\ [\xi-?\mathrm{NumericQ}, \eta-?\mathrm{NumericQ}, \mathrm{n}-\mathrm{Interger?Positive}]: =$

$\sum_{i=1}^{n} a[i]\ \psi\ [i]\ [\xi]\ Exp\ [-\mu[i]^2\eta]$

Where $a[i\ -] := a[i] = -\ \frac{\psi\ [i]'}{\mu[i]^2}[1]$

$\mu\ [i\ -] := \frac{\nu[i]}{\sqrt{2}}$

Asymptotic formulae for the eigenvalues and eigenfunctions are given by Sellers [6] and by direct numerical computation by using some built-in *Mathematica* functions. The eigenvalues are computed once and stored in the memory of the rule,

$\nu[i\ -] :=$

$\nu[i\ -] = \frac{\mathrm{x}}{.\mathrm{FindRoot}}$ [Exp [-x/2] Hypergeometric1F1$[\frac{1}{4}(2-x), 1, x] == 0, \{x, 4\{i-1, i\}\}$,

AccuracyGoal →5]

The eigenfunctions are computed by,

$\psi\ [i\ -][\xi\ -] :=$

$C1\ [i\ -]\mathrm{Hypergeometric1F1}\left[\frac{2-\nu[i]}{4}, 1, \nu[i]\xi^2\right]\ \mathrm{Exp}\ [-\frac{\nu[i]}{2}\xi^2]$

Where

$C1\ [i\ -] := C1[i] =$

$$\frac{1}{\mathrm{sqrt}\ [2\mathrm{NIntegrate}[\mathrm{Evaluate}[e^{-\xi^{2\nu[i]\xi\,(1-\xi^2)}}\mathrm{Hypergeometric\ 1F1}[\frac{1\ ([2-\nu[i]\ ,1,\ \xi^2\nu[i]^2]\ ), \{\xi, 0, 1\}]]}{4}}$$

The first 12 roots coincide with these reported [7] and reprinted in [5],

Table [ν[i], { *i*, 12}

{ 1.6816, 5.66986, 9.66824, 13.6677, 17.6674, 21.6672, 25.6671, 29.667, 33.667, 37.6669, 41.6669, 45.6669}

The first 12 normalization constants are

Table [C1[i], { *i*, 12}]

{1.25857, 1.29773, 1.3011, 1.30202, 1.30239, 1.30257, 1.30268, 1.30275, 1.30279, 1.30282, 1.30284, 1.30286}

The dimensionless temperature at ξ from o to step 0.1 and $\eta = 0.2$ calculated by using 12 terms are :

Table [θ [ξ, 0.2, 12],{ξ, o, 1, 0.1}]

{0.89305, 0.88762, 0.87141, 0.844723, 0.808234, 0.763144, 0.711233, 0.65478, 0.596361, 0.538594, 0}

The plot of the temperature field is

Plot3D[[θ [ξ,*η*, 12] , { *η*, 0.01, 1.5, } , {ξ,*0*, 1},

AxesLabel1→ {ξ, *η*, θ}, Shading→ False Boxed →False];

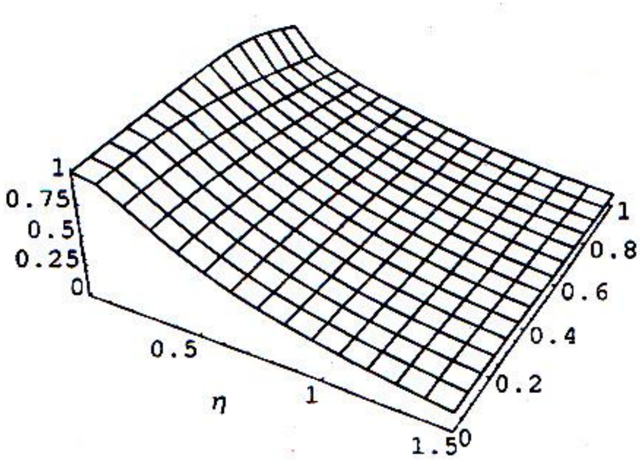

Figure 1: Temperature distribution θ [ξ, η] inside a tube

The solution for average temperature is programmed as:

θav [(0|0., −])] ≔ 1

θav[η−? NumericQ, n − Interger? P{ositive]: = 2 $\sum_{i=1}^{n} a[i]^2\, Exp[-\mu[i]^2\eta]$

The average temperature**s** at η from 0 to 1 step 0.05 calculated by using 12 terms are:

Table [θav [η, 12] , { η, 0, 1, 0.05}

{1, 1.7471, 1.60224, 1.48233, 1.37652, 1.2805, 1.19217, 1.11039, 1.03442, 0.963731, 0.897916, 0.836614, 0.779505, 0.726298, 0.676724, 0.630535, 0.587499, 0.5474, 0.510038, 0.475226, 0.442791}

The plot of the average temperature is

Plot [θa*v* [η, 12], {η, 0, 1.5} Plot Range → {0, 1}, AxesLabel→ { η,[θa*v*}];

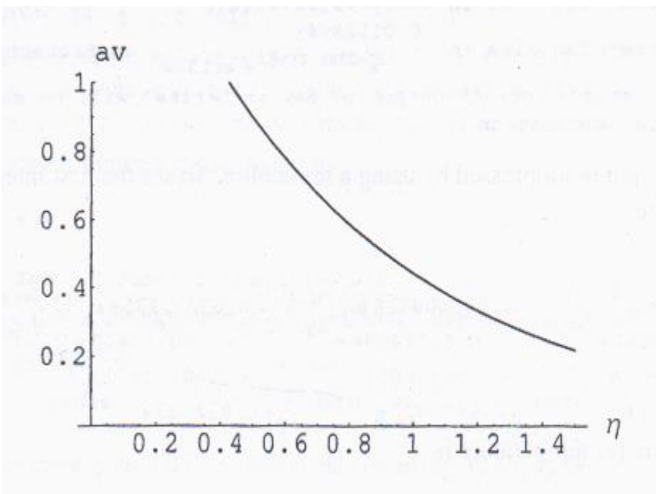

Figure 2: Average temperature θa*v* [η] of a tube

The solution for local Nusselt number is programmed as:

Nu [(0|0., −])] : = ∞

$Nu\ [\ \eta\text{–? NumericQ , n} \quad \text{Integer? Positive}] := \frac{\sum_{i=1}^{n}(a[i]\mu[i]^2 Exp[-\eta\mu[i]^2]}{2\sum_{i=1}^{n}(a[i]^2\ Exp[-\eta\mu[i]^2]}$

The local Nusselt numbers at η from 0 to 0.7 step 0.05 are ;

Table[Nu[η, 12], {η, 0, 0.7, 0.05}]

{∞, 0.948365, 0.808459, 0.754517, 0.729717, 0.717885, 0.712202, 0.709469, 0.708156, 0.707525, 0.707221, 0.707075, 0.707005, 0.706972, 0.706956}

The plot of the Nusselt number is:

Plot[Evaluate { Nu [η, 12], { η, 0.001, 0.1}

AxesLabel → { η, Nu}, PlaoRange - > {2, 6}];

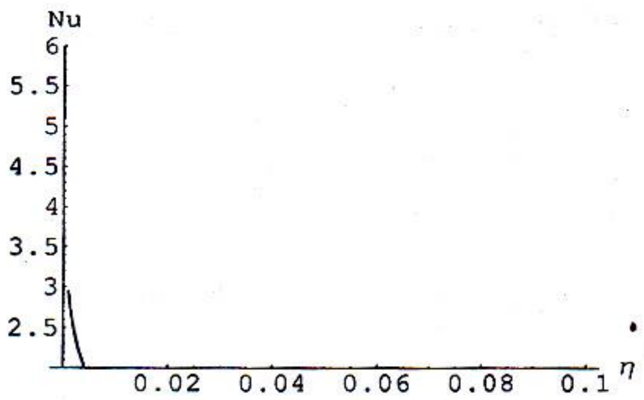

Figure 3: Nusselt number Nu[η] of a tube

4 Generalized Integral Transform Technique, GITT

We solve here the problem from the Eq. (1) using the normalized eigenfunctions by Eqs. (23) –(25)

$\psi_1[\xi] + \mu_i^2 w\ [\xi]\psi_i[\xi] = 0$ (23)

$\psi_1'[0] = 0$ (24)

$\psi_i[1] == 0$ (25)

And for the case w[ξ] = $\sqrt{2}\cos[\mu_i\ \xi]$ (26)

$\mu_i = \left(i - \frac{1}{2}\right)\pi$ (27)

Now to transform Eq. (1) into an ordinary differential equation for the integral transform we use identity equation Eq.(12), the eigenvalue problem did not correspond to the space operator, instead of Eq. (13) we obtain

$\int_0^1 w[\xi]\psi_i[\xi]\ \frac{\partial\theta}{\partial\eta}\ [\xi,\eta]d\xi - \int_0^1 \psi_i[\xi]\frac{1}{\xi}\ \frac{\partial\theta}{\partial\xi}\ [\xi,\eta]d\xi + \mu_i^2\ \theta_i[\eta] = 0$ (28)

The temperature derivatives in the above integrals according to generalized integral transform technique GITT, are removed by using the inversion formula, Eq.(10),

$\sum_{j=1}^{n} w[i,j]\theta_j'\ [\eta] + \mu_i^2\theta_i[\eta] = \sum_{j=1}^{n} v[\ i,j]\theta_j\ [\eta]$ (29)

Where

$w[i,j] = \int_0^1 w[\xi]\psi_i[\xi]\psi_j\ [\xi]d\xi$ (30)

v $[i,j] = \int_0^1 \frac{1}{\xi} \psi_i[\xi]\psi_j' \ [\xi]d\xi$ (31)

The entrance condition Eqs. (1) is transformed as

θ_i [0] = = - $\frac{1}{\mu_i}(-1)^i\sqrt{2}$ (32)

Moreover, introducing Eqs.(3) and (26) into Eq. (30) we obtain

$W[i,j] = \frac{4}{3} + \frac{1}{\mu_i^2}, \quad W[i,j] = -\frac{16\,(-1)^{i+j}}{(\mu_i^2-\mu_j^2)}\mu_i\,\mu_j$ (33)

Introducing Eq.(3) and Eq.(26) into Eq.(30) we obtain ,

$V[i,j] =$ SinIntegral $[2\,\mu_i]\mu_i$ (34)

$V[i,j] =$ (SinIntegral)$[\mu_i - \mu_j]$ − SinIntegral $[\mu_i + \mu_j])\mu_j$

Eq. (29) can be rewritten as in the matrix

$[W]\,\{\theta\}' + [M]\{\theta\} = \{V\}\{\theta\}$ (35)

Where the elements of the matrix [W] and [V] are from Eq.(33) and (34); [M] represent the diagonal matrix formed by the eigenvalues Eq.(27). Furthermore for specified truncation number *n* the matrix system Eq.(35) for the initial conditions Eq.(32) could be solved. Henceforth, once the integral transform $\theta_i[\eta]$are found, the desired temperature θ[ξ,η]

is computed by the inversion formula from Eq.(36)

$\theta[\xi,\eta] = \sum_{i=1}^{n}\theta_i[\eta]\psi_i\,[\xi]$ (36)

Hence the average temperature θav $[\eta]$ and Nusselt number Nu $[\eta]$ are determined by using Eqs.(4)-(5).

5 Calculation and Obtain New Results

The elements of the matrices [V] and [W] from Eqs.(34) and (33) are computed by

V[i−, j −) ≔ If [i === j, −SinIntegral $[2\,\mu\,[i]]\mu[i]$,

(SinIntegral [μ[i] − μ[j]] − SinIntegral [μ[i] + μ[j]]μ[j]

W[i−, j −) ≔ If [i === j, $\frac{4}{3} + \frac{1}{\mu[i]^2}$, $-16\,(-1)^{i+j}\mu[i]\,\mu[j]/\{\,\mu[i]^2 - \mu[j]^2\}^2$

With the eigenvalues

$\mu\,[i\,-]:= (\,i - 0.5)\pi$

The elements of the matrix [M] are computed by

M ≔ Diagonal Matrix [Table [μ[i]^2 , { i, n}]]

The matrix equation (35) could be written as $\{\theta\}'$= [A] $\{\theta\}$, where

A: = Inverse[Table [W[i, j], {i, n}, { j, n}]]

(Table [V[i, j], {i, j}, {j, n}] − M)

The initial condition Eq. (32) could be written as $\theta[0] = \{f\}$ where

f ≔ Table $[(-1)^{i+1}]$ Sqrt $\frac{[2]}{\mu[i]}, \{i,n\}]$

The hybrid solution of the matrix system {θ}′ = [A] {θ} Subject to {θ[0]} = $\{f\}$

myMatrixDSolve [A-?MatrixQ, f-Vectpor Q, x-] := Module [{ev, ef},

{ev, ef} = Eigensystem[Transpose [A]]; Inverse [ef].{Exp[ev x] (ef.f)}]

Again we assume the truncation number n = 30;

Tthe final integral transforms are calculated by,

Evaluate [Table[θ[i][η –], {i, n}]] = myMatrixDSolve [A, f, η]; 0.00213238\

Set : :write: Tag Plus in ⅇ ^(-3181.17\η_(-+)) is protected

0.000600726\ⅇ^«1» + « 8 « +

« 19 «\ⅇ^«1« + 0.832333\ⅇ^«1«

Set : : “write” : Tag Plus in <<21<<+ 0.0176316\ ⅇ^−«19«\η_ is protected

Set : : “write” : Tag Plus in 0.0112864\ ⅇ^−3181.17\η_ + <<13<< is protected

General :: “stop” : further output of Set :: “write” will be suppressed during this calculation

Now the output is suppressed by using a semicolon. Hence, in the first integral transform we enter,

θ[1] [η]

$$0.00213238\, e^{-3181.17\eta} + 0.000600726\, e^{-1392.4\eta} + 0.00072308 e^{-969.697\,\eta} + 0.000610281\, e^{-756.088\eta} + 0.00084094 e^{-586.761\eta} + 0.00117407 e^{-439.201\eta} + 0.000174069\, e^{-312.979\eta} + 0.00280097\, e^{-208.091\eta} + 0.000510477\, e^{-124.537\eta} + 0.0115092\, e^{-62.3166\eta} + 0.0407425\, e^{-21.4315\eta} + 0.832333 e^{-1.88518\,\eta}$$

Moreover, the rule for temperature is,

θ[(–, 0|0)] := 1

θ[(1|1., –)] := 0

θ[ξ, η_-]:=Sum [ψ [i][ξ]θ [i][η], { i, 1, n}

where the normalized eigenfunctions are

ψ [i_-][x_-]:=√(2) cos [μ[i]x]

The dimensional temperature at ξ as 0 to 1 step 0.1 and η = 0.2 are obtained by using 12 terms as:

{0.701236, 0.689342, 0.654387, 0.59856, 0.525465, 0.439883, 0.347286, 0.253112, 0.162, 0.0771591, 0}

For the same ξ and the η the GITT solution by using 30 terms gives

Table [θ [ξ, 0, 2], {ξ, 0, 1, 0.1}]

{ 0.819547, 0.776076, 0.723038, 0.651355, 0.563554, 0.463013, 0.353175, 0.237588, 0.119284, 0}

The rule for average temperature is,

θav = [{0 | 0,]}:=1

θav = [η_-]:=2Sum [W [i] θ [i][η], { i, 1, n}

where

$$W[i\_] := \frac{2\sqrt{2}}{\mu[i]^4}\left(6 + 6\,(-1)^i \mu[i] + \mu\,[i]^2\right.$$

The dimensionless average temperatures at η from 0 to 1 step 0.05 are obtained by using 12 terms, we obtain

{1, 0.716118, 0.578787, 0.476694, 0.395299, 0.328676, 0.27368, 0.227794, 0.18971, 0.158003, 0.131599, 0.328676, 0.273586, 0.227794, 0.18971, 0.158003, 0.131599, 0.109608. 0.0912927, 0.0760678, 0.063332, 0.0527493, 0.043935, 0.0365935, 0.0304788, 0.0253858, 0.0211439 }

The Nusselt number is calculated by

Nu[(0|0,) := ∞

Nu[η_-] :=sqrt [2] Sum [$(-1)^{i\theta}$ [i] [η] μ [i], {i, n}] / θav [η]

The local Nusselt numbers at η from 0 to 0.7 *step* 0.05 are obtained using 12 terms

{ ∞, 2.32014, 2.00231, 1.89597, 1.85499, 1.83888, 1.83252, 1.83002, 1.82904, 1.82865,
1.8285, 1.82844, 1.82841, 1.8284, 1.8284}

Now for the same η the GITT solution also by using 30 terms gives,

Table [Nu[η], { η, 0, 0.7, 0.05}]

$$\begin{Bmatrix} \infty, 2.72509, 2.47296, 2.38973, 2.35912, 2.34824, 2.34479, 2.34411, 2.34441, 2.34504, 2.34575, \\ 2.34645, 2.3471, 2.34771, 2.34827 \end{Bmatrix}$$

The Bessel functions, which correspond to slug flow in tube, will give better convergence of the GITT solution. Now the result in our studies demonstrate that even the trigonometric functions, which correspond to slug flow in parallel plate channel could be used.

Conclusions and discussions

We have demonstrated of the above studies the power of the generalized integral transform technique–which is a typical hybrid method. The present text is adapted using original notebooks documents [3] – where all steps of the solutions statements of the problem, nondimensionalizing, eigenproblem, integral transform pair, transforming to ordinary differential equation and solution of computational rules and results are made by using *Mathematica* [3]. Furthermore, hybrid computation, which combines the power of numerical approach as well as analytical approach is very promising area for research and applications (www.begellhouse.com)

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